Exam 2
Chapter 3 and 4.1-4.5

Short Answer

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Indicate your final answer with a $\boxed{\text{box}}$.

1. Find the derivative of the given function.

(a) (4 points) $f(t) = \sin(t^2 - 1)$.

(b) (6 points) $g(x) = \left(\frac{x^3}{\ln x}\right)^2$.

2. (10 points) The position at time $t \geq 0$ of a particle moving along a horizontal coordinate line is given by

$$s = 20\cos(t + \pi/2).$$

Find the particle's starting position, furthest distance left and right, and its velocity,speed and acceleration.

3. The demand equation for a quantity q of a product at price p, in dollars, is $p = -5q + 5000$. Companies producing the product report the cost, C, in dollars, to produce a quantity q is $C = 10q + 5$.

(a) (2 points) Find the revenue function R as function of quantity, q. (<u>Hint:</u> Revenue=$R = pq$.)

(b) (2 points) Find the profit function P. (<u>Hint:</u> Profit=$P = R - C$)

(c) (6 points) For what quantity q will profit be maximized? What is the maximum profit? (You need to plug in the value, but do not worry about multiplying everything out. Make sure to give units.)

4. (10 points) Consider the function $f(x) = x^2 + \frac{2}{x} = \frac{x^3+2}{x}$. Find all critical and inflection points. Identify the behavior of f on the interval and which critical points correspond to local extrema. For a bonus 5 points identify the asymptotes of f and graph it.

5. Find the desired limits.

(a) (5 points) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$.

(b) (5 points) $\lim_{x\to 0^+} \left(1+\frac{1}{x}\right)^x$.

Bonus Question: (10 points) Allisha Gray is getting some Gatorade after a hard fought victory in the NCAA tournament. Assuming that the Gatorade cooler is a right circular cylinder, the volume V of the liquid remaining in the cooler is given by

$$V = \pi r^2 h \ \ \text{in}^3$$

where h is the height and r is the radius. Furthermore, assume that the radius is a constant of 8 inches. Find dV/dt if the height is decreasing at a constant rate of $\frac{0.1}{\pi}$ inches per second. (Make sure to give units.)